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Pearson Edexcel Level 3 GCE

Centre Number | Candidate Number

Further Mathematics

Advanced
Further Mathematics Option 1
Paper 3: Further Mechanics 1
Further Mathematics Option 2
Paper 4: Further Mechanics 1

Sample Assessment Material for first teaching September 2017

Time: 1 hour 30 minutes

Paper Reference
9FM0/3C
9FM0/4C

You must have:
Mathematical Formulae and Statistical Tables, calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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Answer ALL questions. Write your answers in the spaces provided.

Unless otherwise indicated, whenever a numerical value of g is required, take $g = 9.8\,\text{m s}^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

1. A particle P of mass 0.5 kg is moving with velocity $(4\mathbf{i} + \mathbf{j})\,\text{m s}^{-1}$ when it receives an impulse $(2\mathbf{i} - \mathbf{j})\,\text{N s}$.

Show that the kinetic energy gained by P as a result of the impulse is 12 J.

(6)

the kinetic energy gained can be calculated using $E.K.E_{final} - E.K.E_{initial}$

...let's use the vector version of the Impulse-momentum formula: $I = m(\underline{v} - \underline{u})$

Impulse (Ns), mass (kg), final velocity (ms⁻¹), initial velocity (ms⁻¹)

to get $\underline{v}_{after}$ first:

$$\begin{pmatrix}2\\-1\end{pmatrix} = 0.5\left(\begin{pmatrix}a\\b\end{pmatrix} - \begin{pmatrix}4\\1\end{pmatrix}\right)$$

expand the brackets

$$\begin{pmatrix}2\\-1\end{pmatrix} = 0.5\begin{pmatrix}a-4\\b-1\end{pmatrix}$$

×2 ×2

$$\begin{pmatrix}4\\-2\end{pmatrix} = \begin{pmatrix}a-4\\b-1\end{pmatrix}$$

equate vector components

...$\underline{i}$:

$a - 4 = 4$

$\Rightarrow a = 8$

...$\underline{j}$:

$-2 = b - 1$

$\Rightarrow b = -1$

$$\therefore v = \begin{pmatrix}8\\-1\end{pmatrix}\text{ms}^{-1}$$

↳ now can work out the K.E gained:

formula: $E.K.E_{final} - E.K.E_{gained}$

$$= \frac{1}{2}m(v^2 - u^2)$$

↳ formula requires vectors to be scalars

hence Pythagorise:

$u = \begin{pmatrix}4\\1\end{pmatrix}$

$|u| = \sqrt{(4)^2 + (1)^2}$

$= \sqrt{17}$

$v = \begin{pmatrix}8\\-1\end{pmatrix}$

$|v| = \sqrt{(8)^2 + (-1)^2}$

$= \sqrt{65}$

subbing scalars into the formula

Question 1 continued

$$= \frac{1}{2}\left(\frac{1}{2}\right)\left(\left(\sqrt{65}\right)^2 - \left(\sqrt{17}\right)^2\right)$$

$$= \frac{1}{4}(65 - 17)$$

$$= \frac{1}{4}(48) = \boxed{12\text{J as required}}$$

(Total for Question 1 is 6 marks)

2. A parcel of mass 5 kg is projected with speed 8 m s^{-1} up a line of greatest slope of a fixed rough inclined ramp.

The ramp is inclined at angle α to the horizontal, where $\sin\alpha = \frac{1}{7}$

The parcel is projected from the point A on the ramp and comes to instantaneous rest at the point B on the ramp, where $AB = 14$ m.

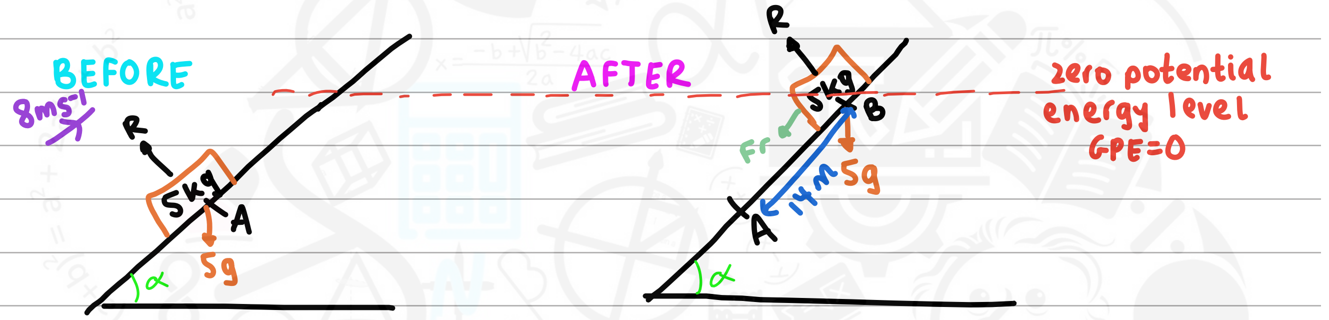
The coefficient of friction between the parcel and the ramp is μ.

In a model of the parcel's motion, the parcel is treated as a particle.

(a) Use the work-energy principle to find the value of μ. **(5)**

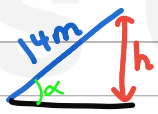
(b) Suggest one way in which the model could be refined to make it more realistic. **(1)**

(a) Let's illustrate the above information diagrammatically - best to draw two diagrams - one for before and one for after the particle travels a distance AB = 14m up the plane, labelling the respective energies



· E.K.E

· G.P.E → but since the parcel is going up an inclined plane we need the perp. distance of 14m
↳ hence constructing a suitable trig triangle



$\Rightarrow h = 14\sin\alpha$

and given that $\sin\alpha = \frac{1}{7}$

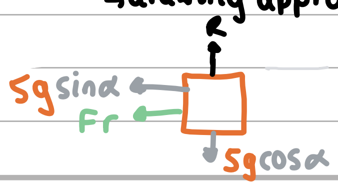
$\Rightarrow h = 14\left(\frac{1}{7}\right) = 2$

· W.d by friction

formula: $Fr = \mu R$ → resultant force

Fr ↓ friction, J; μ ↓ coefficient of friction

but need to resolve to get this resultant force
↳ drawing appropriate force diagram



R(↑): $R = 5g\cos\alpha$

and we can get $\cos\alpha$ from $\sin\alpha = 1/7$ using the trig identity: $\sin^2\alpha + \cos^2\alpha \equiv 1$ rearranged

identity: $\sin^2\alpha + \cos^2\alpha \equiv 1$

$$\Rightarrow \cos^2\alpha = 1 - \sin^2\alpha$$
$$= 1 - (1/7)^2$$
$$= \frac{48}{49}$$

$$\therefore \cos\alpha = \sqrt{\frac{48}{49}} = \frac{4\sqrt{3}}{7}$$

subbing it into our R

$$R = 5g\left(\frac{4\sqrt{3}}{7}\right)$$
$$= \frac{20\sqrt{3}}{7}g$$

$$\Rightarrow Fr = \mu \frac{20\sqrt{3}}{7}g$$

...subbing above into our work-energy principle; this means total energies before = total energies after

sub above into the work-energy principle : includes dissipative forces)

$$\underbrace{\text{w.d in}}_{\text{n/a}} + \underbrace{K.E_i}_{\text{kinetic initial}} + \underbrace{G.P.E_i}_{\text{gravitational potential initial}} + \underbrace{E.P.E_i}_{\text{elastic potential initial}} = \underbrace{K.E_f}_{\text{kinetic energy final}} + \underbrace{G.P.E_f}_{\text{gravitational potential final}} + \underbrace{E.P.E_f}_{\text{elastic potential final}} + \text{w.d against friction}$$

$$\frac{1}{2}mu^2 + mgh_1 + \frac{\lambda x^2}{2l} = \frac{1}{2}mv^2 + mgh_2 + \frac{\lambda x^2}{2l} + Fr \times d$$

want to solve for!

$$\frac{1}{2}(5)(8)^2 + 0 + 0 = 0 + 5(g)(2) + \mu\left(\frac{20\sqrt{7}}{3}g\right)(14)$$

expand above

$$160 = 10g + 40\sqrt{3}g\mu$$

solve for 'μ'

$$40\sqrt{3}g\mu = 62$$

$$\Rightarrow \mu = \frac{62}{40\sqrt{3}g}$$
$$= 0.09131\ldots$$
$$\boxed{= 0.091\ (3\text{s.f})}$$

Question 2 continued

(b) looking back at Mechanics Yr 1 Chp 8:

- parcel shouldn't be modelled as a particle $\therefore$ air resistance could be taken account of
- take into account dimensions/uniformity of the parcel

(Total for Question 2 is 6 marks)

3. A particle of mass m kg lies on a smooth horizontal surface.

Initially the particle is at rest at a point O between two fixed parallel vertical walls.

The point O is equidistant from the two walls and the walls are 4 m apart.

At time $t = 0$ the particle is projected from O with speed $u\,\text{m s}^{-1}$ in a direction perpendicular to the walls.

The coefficient of restitution between the particle and each wall is $\frac{3}{4}$

The magnitude of the impulse on the particle due to the first impact with a wall is λmu N s.

(a) Find the value of λ.

(3)

The particle returns to O, having bounced off each wall once, at time $t = 7$ seconds.

(b) Find the value of u.

(5)

(a) illustrating the first impact of the ball with the wall diagrammatically: label the respective speeds, direction of impulse

← from NEL rearranged:

formula: $e = \frac{v}{u}$

solve for 'v'

$v = eu$

now that we have the velocity before and the velocity after, we can work out the impulse:

formula: $I = m(v - u)$

sub into above:

$I = m\left(\frac{3}{4}u - (-u)\right)$

$= m\left(\frac{7}{4}u\right) = \frac{7}{4}mu$ (Ns)

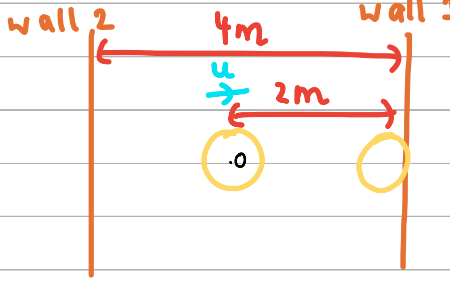
$\boxed{\therefore \lambda = 7/4}$

(b) notice we're dealing with successive collisions in 1D - specifically DISTANCE-question

↳ let's break the ball's journey into 3 sections and, because given info on time, evaluate $\text{speed} = \frac{\text{distance}}{\text{time}}$ at each section

Question 3 continued

① from O to wall 1

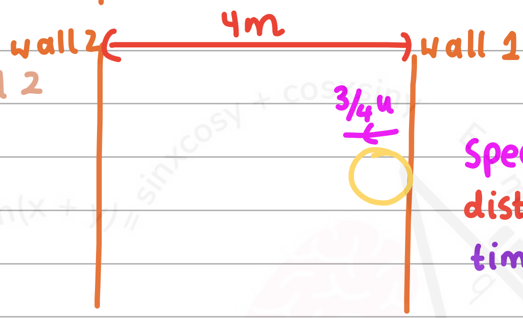


speed $= u$

distance $= 2$

time $= \frac{\text{distance}}{\text{speed}} = \frac{2}{u}$

② wall 1 to wall 2



Speed $= 3/4u$

distance $= 4$

time $= \frac{4}{3/4u} = \frac{16}{3u}$

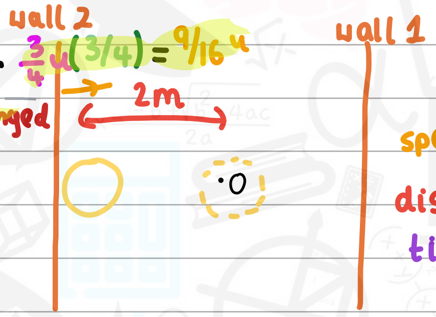
③ wall 2 back to O

$\frac{3}{4}u(3/4) = 9/16u$

from NEL rearranged

formula: $e = \frac{v}{u}$

$\Rightarrow v = eu$



speed $= \frac{9}{16}u$

distance $= 2$

time $= \frac{2}{9/16u} = \frac{32}{9u}$

now finally can use the fact that the particle's total journey time is 7s - hence summing the times from each section

$\therefore t_1 + t_2 + t_3 = 7$

$\frac{2}{u} + \frac{16}{3u} + \frac{32}{9u} = 7$

need to solve for 'u'

getting common denominator:

$\frac{9(2)}{9u} + \frac{3(16)}{9u} + \frac{32}{9u} = 7$

collect like terms and solve for 'u'

$\frac{98}{9u} = 7$

$\times 9u \qquad \times 9u$

$63u = 98$

$\div 63 \qquad \div 63$

$\boxed{u = 14/9}$

(Total for Question 3 is 8 marks)

4.

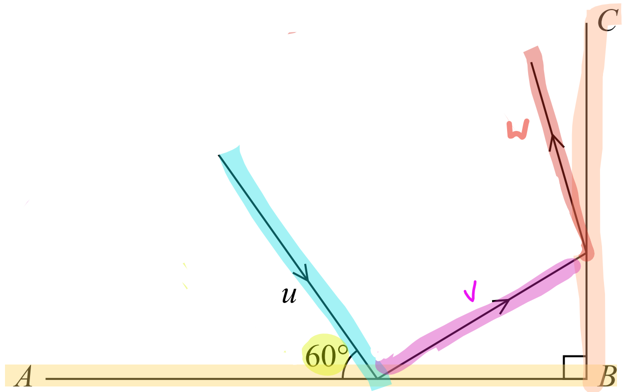


Figure 1

Figure 1 represents the plan view of part of a horizontal floor, where AB and BC are perpendicular vertical walls.

The floor and the walls are modelled as smooth.

A ball is projected along the floor towards AB with speed $u\,\mathrm{m\,s^{-1}}$ on a path at an angle of 60° to AB. The ball hits AB and then hits BC.

The ball is modelled as a particle.

The coefficient of restitution between the ball and wall AB is $\dfrac{1}{\sqrt{3}}$

The coefficient of restitution between the ball and wall BC is $\sqrt{\dfrac{2}{5}}$

(a) Show that, using this model, the final kinetic energy of the ball is 35% of the initial kinetic energy of the ball.

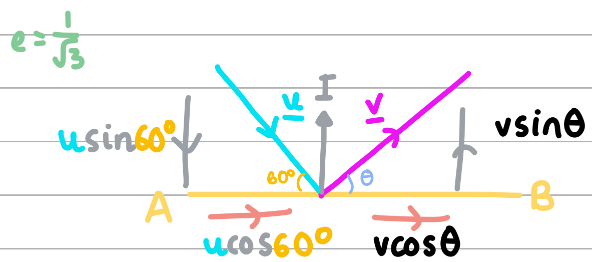
(8)

(b) In reality the floor and the walls may not be smooth. What effect will the model have had on the calculation of the percentage of kinetic energy remaining?

(1)

(a) notice we're dealing with a successive oblique collisions (with fixed surfaces) question - need to work out our final velocity 'w' first in order to then find $E.K.E_{final}$

...first concentrating on first collision between the ball and the wall AB



... perp. components:

Impulse always acts perpendicular to the fixed surface ∴ NEL rearranged applies

formula: $e=\frac{v}{u}$ $\Rightarrow v=eu$

Question 4 continued

$\Rightarrow v\sin\theta = \frac{1}{\sqrt{3}} u\sin 60°$

hence using $\sin 60° = \sqrt{3}/2$

$\Rightarrow v\sin\theta = \frac{u}{2}$

.:. parallel comps:

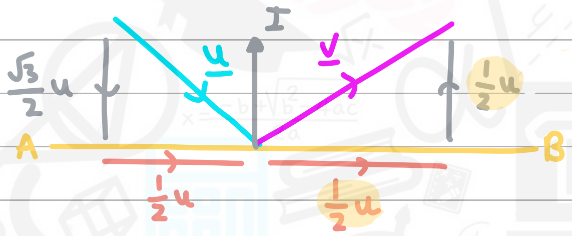
no impulse acting parallel to the fixed surface ∴ no change

$v\cos\theta = u\cos 60°$

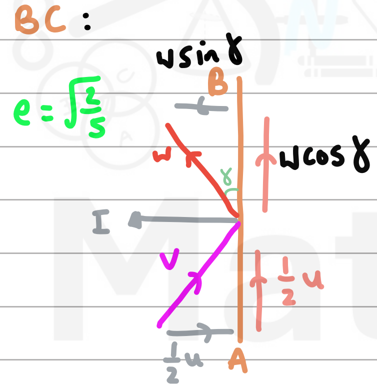
hence using $\cos 60° = 1/2$

$\Rightarrow v\cos\theta = 1/2$

...hence populating this onto our diagram:



...next looking at the second collision - the one between the ball and the wall BC :



... perp. components :

Impulse acts perpendicular to the fixed surface ∴ using NEL rearranged

formula: $e = \frac{v}{u}$

$\Rightarrow v = e\,w$

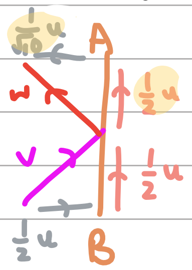
$\Rightarrow u\sin\gamma = \sqrt{\frac{2}{5}}\left(\frac{1}{2}u\right) = \frac{1}{\sqrt{10}}u$

...parallel component:

↳ no Impulse ∴ no impact

$\Rightarrow w\cos\gamma = \frac{1}{2}u$

...let's populate this onto our diagram:



$\therefore w = \begin{pmatrix} -\frac{1}{\sqrt{10}}u \\ \frac{1}{2}u \end{pmatrix}$

Question 4 continued

now that we've got the parallel and perpendicular components of u, we have enough information to work out the kinetic energy before and after the two successive collisions

formula: $\frac{1}{2}mv^2$

... K.E initial :

$= \frac{1}{2}mu^2$

... K.E final :

$\frac{1}{2}m\left(-\frac{1}{\sqrt{10}}u\right)^2 + \frac{1}{2}m\left(\frac{1}{2}u\right)^2$

$= \frac{1}{2}mu^2\left(\frac{1}{10} + \frac{1}{4}\right)$

$= \frac{7}{40}mu^2$

∴ as a fraction of the K.E initial : $\dfrac{\frac{7}{40}m\cancel{u}^2}{\frac{1}{2}m\cancel{u}^2} = \frac{7}{20} = \boxed{35\%}$

(b) % will be smaller as more of the energy will be lost to friction

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Question 4 continued

(Total for Question 4 is 9 marks)

5. A car of mass 600 kg is moving along a straight horizontal road.

At the instant when the speed of the car is v m s^{-1}, the resistance to the motion of the car is modelled as a force of magnitude $(200 + 2v)$ N.

The engine of the car is working at a constant rate of 12 kW.

(a) Find the acceleration of the car at the instant when $v = 20$

(4)

Later on the car is moving up a straight road inclined at an angle θ to the horizontal,

where $\sin\theta = \frac{1}{14}$

At the instant when the speed of the car is v m s^{-1}, the resistance to the motion of the car from non-gravitational forces is modelled as a force of magnitude $(200 + 2v)$ N.

The engine is again working at a constant rate of 12 kW.

At the instant when the car has speed w m s^{-1}, the car is decelerating at 0.05 m s^{-2}.

(b) Find the value of w.

(5)

(a) illustrating the above information diagrammatically – label the respective speeds, variable resistance and the power rearranged

sub in $V = 20\text{ms}^{-1}$

$\therefore 200 + 2(20)$

$= 240\text{N}$

formula: $P = Fv$ (POWER in Watts, FORCE in Newtons, VELOCITY in ms^{-1})

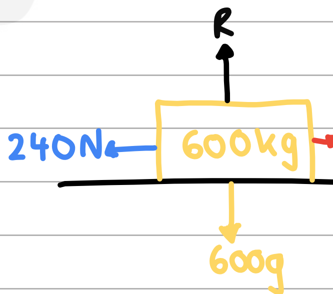
$\Rightarrow F = \frac{P}{v}$

...where:

$12\text{kW} \xrightarrow[\times 1000]{\text{convert to Watts}} 12{,}000\text{W}$

and $v = 20$

NOTE: this could've been a separate line of working but much more time efficient in the exam to label straight onto diagram



$F = \frac{12{,}000}{20} = 600\text{N}$

finding 'acceleration' implies we've got a dynamics problem $\therefore$ using Newton's second law and resolving horizontally:

formula: $\Sigma F = ma$

$R(\rightarrow^{+})$: $600 - 240 = 600a$ ← need to solve!

$\Rightarrow 600a = 360$
$\div 600 \qquad \div 600$

$\Rightarrow a = \frac{360}{600} = \frac{3}{5} = \boxed{0.6\text{ms}^{-1}}$

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Question 5 continued

(b) now the car is moving up an inclined slope - hence drawing the relevant diagram: label the variable resistance and power rearranged

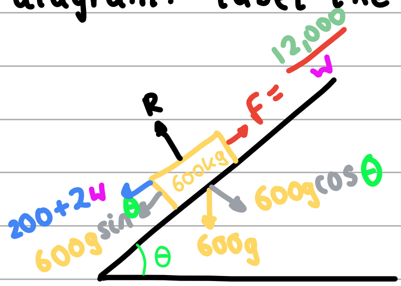
here sub in $v = w$

formula: $P = Fv$

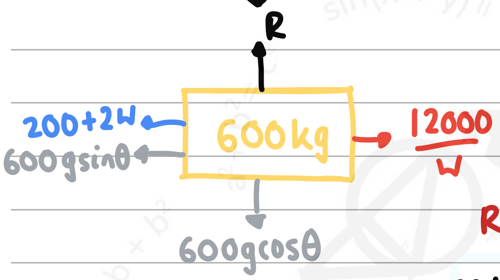
$\Rightarrow F = \frac{P}{v}$

$12\text{kW} \xrightarrow[\times 1000]{\text{convert to Watts}} 12{,}000\text{W}$

and $v = w$



turn the axis



we are given, this time, the (-ve) acceleration of the car, hence treating it as another dynamics problem: applying $\Sigma F = ma$ (Newton's Second Law)

$R(\rightarrow +)$: $\frac{12000}{w} - (200+2w) - 600g\sin\theta = 600(-0.05)$

need to solve for this!

we're given that $\sin\theta = 1/14$, hence subbing this into the above

$$\frac{12{,}000}{w} - 200 - 2w - 600g\left(1/14\right) = -30$$

expand and collect like terms:

$$\frac{12{,}000}{w} - 620 - 2w = -30$$

$\times w$ $\times w$

$$2w^2 + 590w - 12{,}000 = 0$$

solve the above quadratic for 'w':

calc equtn solver or quadratic formula:

$$w = \frac{-590 \pm \sqrt{(590)^2 - 4(2)(-12{,}000)}}{2(2)}$$

$$\Rightarrow w = \frac{-590 \pm \sqrt{444{,}100}}{4} = 19.102\ldots \text{ or } -314.10\ldots$$

$\boxed{\therefore w = 19.1\text{ms}^{-1} \text{ (3 s.f)}}$

(Total for Question 5 is 9 marks)

6. [In this question **i** and **j** are perpendicular unit vectors in a horizontal plane.]

A smooth uniform sphere A has mass $2m$ kg and another smooth uniform sphere B, with the same radius as A, has mass $3m$ kg.

The spheres are moving on a smooth horizontal plane when they collide obliquely.

Immediately before the collision the velocity of A is $(3\mathbf{i} + 3\mathbf{j})\,\mathrm{m\,s^{-1}}$ and the velocity of B is $(-5\mathbf{i} + 2\mathbf{j})\,\mathrm{m\,s^{-1}}$.

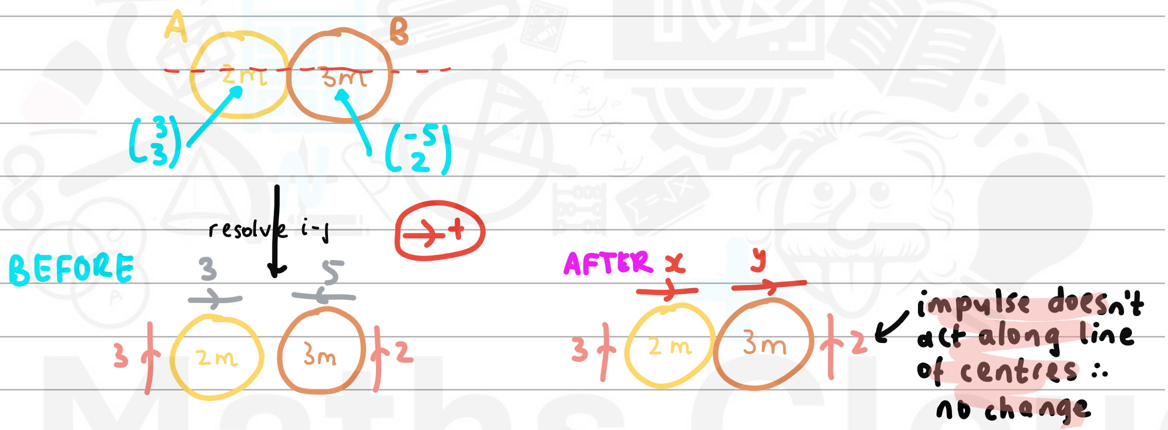
At the instant of collision, the line joining the centres of the spheres is parallel to **i**.

The coefficient of restitution between the spheres is $\frac{1}{4}$

(a) Find the velocity of B immediately after the collision. **(7)**

(b) Find, to the nearest degree, the size of the angle through which the direction of motion of B is deflected as a result of the collision. **(2)**

(a) see now we have an oblique collision between two spheres question - let's illustrate this diagrammatically :



and asked for the parallel and perp. components of V_B :

... perp. :

$= 2$

... parallel : need to find 'y'

∴ standard procedure as for elastic collisions in 1D : PCLM and NEL

... first PCLM i.e total momentum before = total momentum after

formula : $m_A u_A + m_B u_B = m_A V_A + m_B V_B$

Question 6 continued

Subbing into above :

$2\cancel{m}(3) + 3\cancel{m}(-5) = 2\cancel{m}x + 3\cancel{m}y$

$\Rightarrow 2x + 3y = -9$ – ①

...next: NEL (Impact law):

formula: $e = \dfrac{v_B - v_A}{u_A - u_B}$

subbing into above:

$\dfrac{1}{4} = \dfrac{y - x}{3-(-5)}$

$\Rightarrow y - x = 2$ – ②

solve ① and ② simultaneously – need to solve for 'y' so eliminate 'x'

① + 2×②

$2x + 3y = -9$

$+\; -2x + 2y = 4$

$5y = -5$

$\div 5 \qquad \div 5$

$\Rightarrow y = -1$

∴ parallel component of $v_B = -1$

$\boxed{\therefore\ v_B = \begin{pmatrix} -1 \\ 2 \end{pmatrix} \text{ms}^{-1}}$

(b) there are two ways to find the angle of deflection for B :

WAY 1: scalar dot product

formula: $\cos\theta = \dfrac{u \cdot v}{|u||v|}$

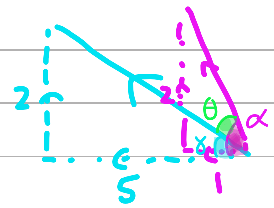
subbing into above

$\cos\theta = \dfrac{\begin{pmatrix} -3 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 2 \end{pmatrix}}{\sqrt{(-3)^2+(2)^2}\sqrt{(-1)^2+(2)^2}}$

expand and evaluate scalar product

$\cos\theta = \dfrac{9}{\sqrt{29}\sqrt{5}} \Rightarrow \theta = \cos^{-1}\left(\dfrac{9}{\sqrt{29}\sqrt{5}}\right)$

WAY 2: using triangles and trig



$\Rightarrow \theta = \alpha - \gamma$

$= \tan^{-1}\left(\frac{2}{1}\right) - \tan^{-1}\left(\frac{2}{5}\right)$

$= 41.6335...$

$\boxed{= 42^\circ (2\text{ s.f})}$

Question 6 continued

$= 41.6335\ldots$

$\boxed{= 42° \text{ (2 s.f)}}$

Question 6 continued

(Total for Question 6 is 9 marks)

7. A particle P of mass m is attached to one end of a light elastic string of natural length a and modulus of elasticity $3mg$.

The other end of the string is attached to a fixed point O on a ceiling.

The particle hangs freely in equilibrium at a distance d vertically below O.

(a) Show that $d = \frac{4}{3}a$.

(3)

The point A is vertically below O such that $OA = 2a$.

The particle is held at rest at A, then released and first comes to instantaneous rest at the point B.

(b) Find, in terms of g, the acceleration of P immediately after it is released from rest.

(3)

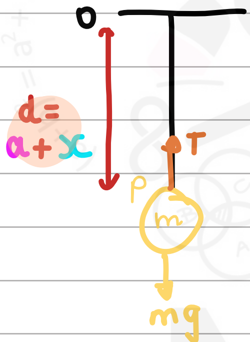
(c) Find, in terms of g and a, the maximum speed attained by P as it moves from A to B.

(5)

(d) Find, in terms of a, the distance OB.

(3)

(a) noticing how the first part of the question refers to an equilibrium problem – let's first draw a detailed diagram:



$l = x$
$\lambda = 3mg$

to prove $d = 4/3\,a$, the question is basically asking us to find the string's extension, x

... hence using the fact it's in equilibrium ∴

forces up = forces down

$\Rightarrow R(\uparrow): T = mg$

and subbing into our formula for elastic strings and springs:

formula: $T = \frac{\lambda x}{l}$ ← solving for this!

$$\cancel{mg} = \frac{3\cancel{mg}\,x}{a}$$

cancel mg's and solve for 'x':

$$\Rightarrow x = \frac{a}{3}$$

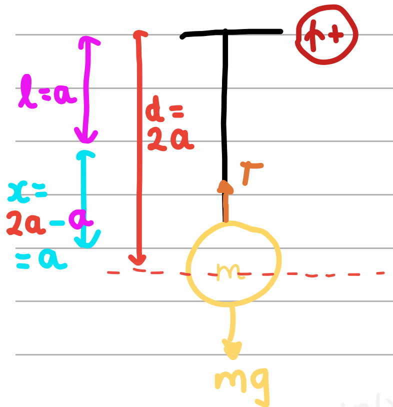
hence subbing into our distance formula

$$d = a + \frac{a}{3}$$

$$\boxed{= \frac{4}{3}a \text{ as required}}$$

Question 7 continued

(b) now that we're asked for the acceleration, we know we're dealing with a DYNAMICS PROBLEM - illustrating this on a detailed diagram:



and using Newton's Second Law to find 'a'

formula: $\Sigma F = ma$ (resolved resultant force, N; mass, kg; acceleration, ms^{-2} ← want to solve!)

$R(\downarrow)$: $T - mg = ma$

but need the value of T, hence subbing into our formula for elastic strings and springs

formula: $T = \dfrac{\lambda x}{l}$ (mod of elasticity; extension, m; natural length, m)

$T = \dfrac{3mg(\cancel{a})}{\cancel{a}} = 3mg$

now let's sub T into our prev. equation

$3\cancel{m}g - \cancel{m}g = \cancel{m}a$

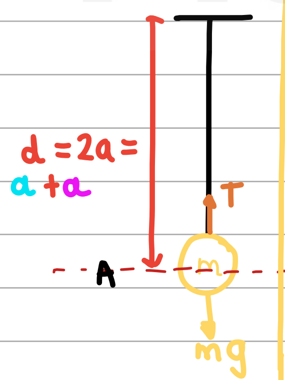
cancel m's and solve for 'a'

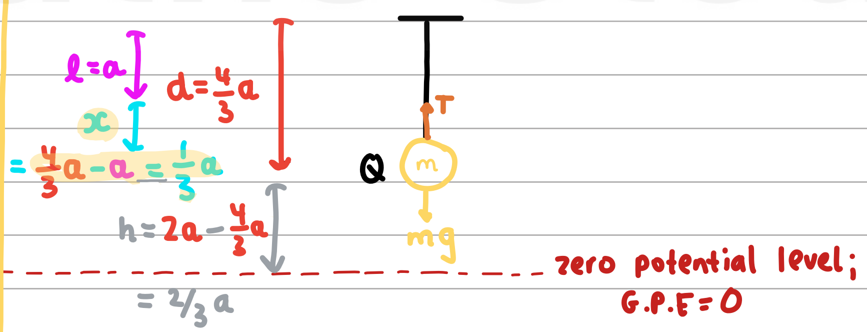
$\Rightarrow \boxed{a = 2g}$

(c) now we're asked to find the max. speed of P as it moves from A to B - know that this happens at some point, call it Q, where the particle is in equilibrium i.e $a=0$

↳ but looking at part (a), the particle is in equilibrium where $d = \frac{4}{3}a$, hence subbing this into the work-energy principle i.e total energies before = total energies after

BEFORE REACHING Q | AFTER REACHING Q





· E.P.E

· K.E
· G.P.E
· E.P.E

Question 7 continued

sub above into the work-energy principle : includes dissipative forces)

$$\underset{n/a}{W.d\ in} + \underset{\text{kinetic initial}}{K.E_i} + \underset{\text{gravitational potential initial}}{G.P.E_i} + \underset{\text{elastic potential initial}}{E.P.E_i} = \underset{\text{kinetic energy final}}{K.E_f} + \underset{\text{gravitational potential final}}{G.P.E_f} + \underset{\text{elastic potential final}}{E.P.E_f} + W.d \text{ against friction}$$

$$\frac{1}{2}mu^2 + mgh_1 + \frac{\lambda x^2}{2l} = \frac{1}{2}mv^2 + mgh_2 + \frac{\lambda x^2}{2l} + F_r \times d$$

want to solve for this!

$$0 + 0 + \frac{3\cancel{m}g(\cancel{a})^2}{2\cancel{a}} = \frac{1}{2}\cancel{m}(v)^2 + \cancel{m}gh + \frac{3\cancel{m}g(\frac{1}{3}\cancel{a})^2}{2(\cancel{a})}$$

cancel m's and simplify:

$$\frac{3}{2}ag = \frac{1}{2}v^2 + \frac{2}{3}ag + \frac{ag}{6}$$

collect like terms

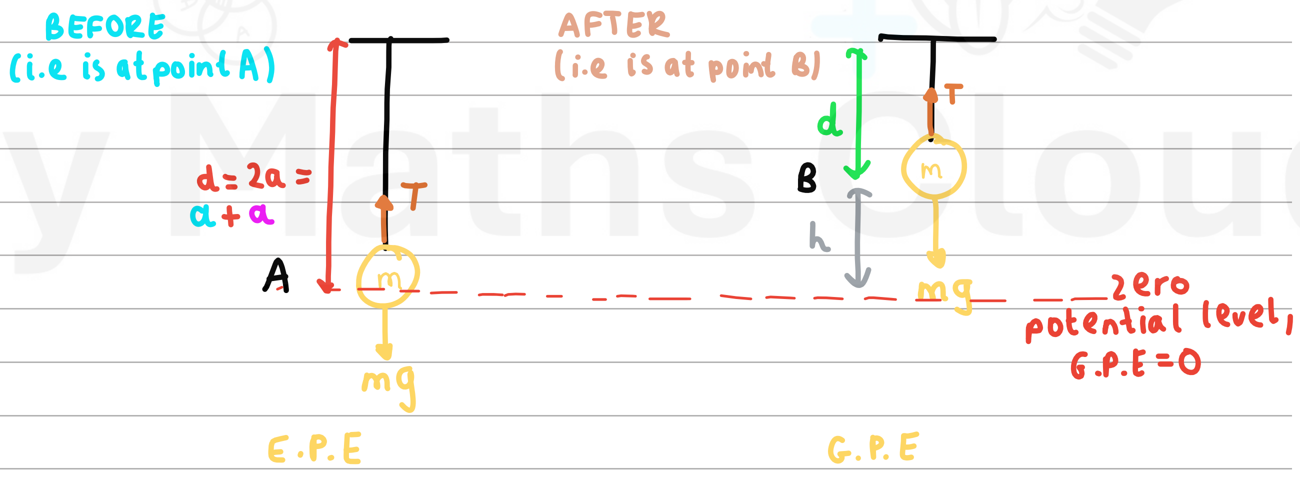
$$\frac{1}{2}v^2 = \frac{2}{3}ag$$

$\times 2$ $\times 2$

$$v^2 = \frac{4}{3}ag$$

$$\Rightarrow \boxed{v = \sqrt{\frac{4}{3}ag}\ ms^{-1}}$$

(d) now we're back to the diagram where we only have A and B - drawing it out and subbing into our work-energy principle - i.e total energies before = total energies after



$$\Rightarrow E.P.E_{lost} = G.P.E_{gained}$$

formula: $\frac{\lambda x^2}{2l} = mgh$ (λ: mod. of elasticity; x: extension, m; l: natural length, m; m: mass, kg; g: grav. field strength; h: height, m)

$$\frac{3\cancel{m}g(\cancel{a})^2}{2(\cancel{a})} = \cancel{m}gh$$

Question 7 continued

expand and simplify

$\frac{3}{2}a = h$

$\therefore d = 2a - \frac{3}{2}a$

$\boxed{= \frac{1}{2}a \text{ m}}$

(Total for Question 7 is 14 marks)

8. A particle P of mass $2m$ and a particle Q of mass $5m$ are moving along the same straight line on a smooth horizontal plane.

They are moving in opposite directions towards each other and collide directly.

Immediately before the collision the speed of P is $2u$ and the speed of Q is u.

The direction of motion of Q is reversed by the collision.

The coefficient of restitution between P and Q is e.

(a) Find the range of possible values of e.

(8)

Given that $e = \frac{1}{3}$

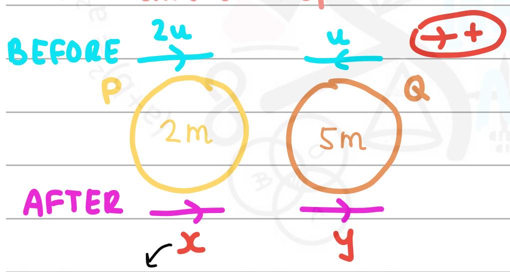
(b) show that the kinetic energy lost in the collision is $\frac{40mu^2}{7}$.

(5)

(c) Without doing any further calculation, state how the amount of kinetic energy lost in the collision would change if $e > \frac{1}{3}$

(1)

(a) realising this is an elastic collisions in 1D question - illustrating the info on a detailed diagram - label respective weights, speeds before and after, and direction of motion



NOTE: realistically P will not keep going right after collision but let's just model it as right to avoid -ves in our working

...and following the standard procedure for elastic collisions in 1D questions:

...using PCLM - means total momentum before = total momentum after:

formula: $m_P u_P + m_Q u_Q = m_P v_P + m_Q v_Q$

subbing into above:

$2\cancel{m}(2u) + 5\cancel{m}(-u) = 2\cancel{m}(x) + 5\cancel{m}(y)$

cancel m's and expand

$2x + 5y = -u$ - ①

...using NEL - impact law:

formula: $e = \frac{\text{speed of separation}}{\text{speed of approach}} = \frac{v_Q - v_P}{u_P - u_Q}$

subbing into above:

$e = \frac{y - x}{2u - (-u)} \Rightarrow y - x = 3eu$ - ②

DO NOT WRITE IN THIS AREA

Question 8 continued

solving ① and ② simultaneously to get 'y' - elim. 'x'

①+2×②

$$\begin{array}{r} 2x+5y=-u \\ + \quad -2x+2y=6eu \\ \hline 7y=-u+6eu \end{array}$$

÷7 ÷7

solve for 'y'

$$y=\frac{u}{7}(6e-1)$$

now we need to identify a source of inequality - the fact that the motion of Q is reversed (which we'd already accounted for in the diagram) implies that $v_Q > 0$

$$\Rightarrow \frac{u}{7}(6e-1) > 0$$

$$\Rightarrow 6e-1 > 0$$

$$\Rightarrow 6e > 1$$

$$\therefore e > 1/6$$

and know $0 \leq e \leq 1$ ∴ the upper bound of e would be 1

$$\boxed{\therefore \frac{1}{6} < e \leq 1}$$

(b) to calculate the kinetic energy lost we use $K.E_{initial} - K.E_{final}$

...first let's calculate $K.E_{initial}$:

formula: $E_k = \frac{1}{2} m v^2$ ← velocity in ms^{-1}, m → mass in kg

$$K.E_{initial} = \frac{1}{2}(2m)(2u)^2 + \frac{1}{2}(5m)(-u)^2$$

$$= 4mu^2 + \frac{5}{2}mu^2$$

$$\boxed{= \frac{13}{2}mu^2}$$

...next, $K.E_{final}$:

but for this need to calculate v_A

hence solving ① and ② simultaneously (from part (a)) - need 'x' so elim. 'y'

5×② - ①

$$\begin{array}{r} -5x+5y=15eu \\ - \quad 2x+5y=-u \\ \hline -7x=15eu-u \end{array}$$

Question 8 continued

solve for x

$\div 7$ $\div 7$

$$x = -\frac{u}{7}(15e+1)$$

and now that we've got x and y, we can sub in $e = 1/3$ into both:

$$V_P = -\frac{u}{7}\left(15\left(\frac{1}{3}\right)+1\right) \qquad V_Q = \frac{u}{7}\left(6\left(\frac{1}{3}\right)-1\right)$$

$$= -\frac{6u}{7} \qquad = \frac{u}{7}$$

$\therefore$ subbing into our formula for K.E:

$$K.E_{final} = \frac{1}{2}(2m)\left(-\frac{6u}{7}\right)^2 + \frac{1}{2}(5m)\left(\frac{u}{7}\right)^2$$

$$= \frac{36}{49}mu^2 + \frac{5}{98}mu^2$$

$$\boxed{= \frac{11}{14}mu^2}$$

$$\therefore E.K_{lost} = \left(\frac{13}{2} - \frac{11}{14}\right)mu^2$$

$$\boxed{= \frac{40}{7}mu^2}$$

(c) if you increase e then this means the collision is more elastic (the value of e is closer to 1) $\therefore$ less energy would be lost than what is calculated in part (b)

Question 8 continued

(Total for Question 8 is 14 marks)

TOTAL FOR PAPER IS 75 MARKS